

A hybrid Approach for Fast Simulation of X-ray Computed Tomography

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Abstract— With the advent of powerful computers the use of Monte Carlo (MC) techniques for radiation transport simulation has become one of the most popular methods for modeling x-ray computed tomography (CT) scanners. One significant problem in the use of MC calculations is the presence of statistical uncertainties in the estimates. A simple but not practical way to decrease statistical uncertainties is to run MC simulations for sufficiently long time (large number of histories) and use efficient variance reduction techniques. In this paper, we propose a hybrid approach combining MC and analytical simulations to speed up x-ray CT modeling. In addition, an analytic method for modeling of beam hardening effect (BHE) in CT is proposed for fast generation of raw data for use in validation of BHE correction algorithms. In this method, the contribution of the primary component to the projections is calculated through analytical simulations using ray-tracing methods whereas the contribution of the scatter component is calculated using pure MC simulations. The final projections are determined by appropriate combination of both simulation results. The hybrid simulator and analytic BHE modelling were validated through comparison with experimental measurements and pure MC simulation of different phantoms performed on clinical CT scanners. In general, there is good agreement between the simulated and measured projections and reconstructed images. This work has demonstrated the feasibility of using the hybrid approach for fast and accurate modeling of CT scanners thus allowing to evaluate the effect of physical, geometrical and other design parameters on CT scanner performance and image quality. The results also validate the proposed method for analytical modeling of BHE which is a useful tool for fast generation of raw CT datasets.

I. INTRODUCTION

X-ray computed tomography (CT) scanners have inherently the tendency to produce physics-related artefacts compared to conventional planar radiography owing to the fact that the images are reconstructed from a large

number of independent detector elements. The simulation of x-ray CT imaging to assess qualitatively and quantitatively the image formation process and interpretation and to assist the development of new detector configurations using deterministic methods and simplifying approximations have been developed mainly to improve speed of operation. Analytical x-ray CT simulators are based on projection ray-tracing methods for the three-dimensional (3-D) calculation of intersections between trajectories of photons emitted from the x-ray tube focal spot toward the detector elements and all voxels or surfaces' equations for each x-ray energy bin since the attenuation coefficients of different materials are energy-dependent [1]. Monte Carlo (MC)-based simulations are based on direct transport of photons and electrons into the materials in a 3-D geometry. One significant problem in the use of MC calculations is the presence of statistical uncertainties (noise) in the estimates [2]. A simple but not practical way to decrease statistical uncertainties is to run MC simulations for sufficiently long time (large number of histories) and use efficient variance reduction techniques. During the last decade, several research groups have investigated the issue of fast simulation of x-ray imaging [3]. It is well known that analytic algorithms based on ray-tracing techniques make it possible to simulate in a short time realistic radiographs. The analytic approach is very fast but actually limited to primary radiation modeling only (i.e., photons that do not interact in the object before being detected). On the other hand, the stochastic nature of involved processes such as x-ray photons generation, interaction with matter and detection makes MC the ideal tool for accurate modeling of x-ray imaging systems.

When a polychromatic x-ray beam passes through matter, low-energy photons are preferentially absorbed, as the linear attenuation coefficient generally decreases with energy. As a result, the beam gradually becomes harder, i.e. its mean energy increases. The harder a beam, the less it is further attenuated. Therefore, the total attenuation is no longer a linear function of absorber thickness. This effect leads to various well-known beam hardening artifacts such as cupping and streak artifacts in reconstructed CT images [4]. Several beam hardening correction (BHC) strategies have been proposed so far. However, most approaches rely on validation studies based on

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accurate modeling of BHE during generation of raw data before the technique can be used with confidence in clinical setting. MC generation of raw data for assessment of correction algorithms' performance is time consuming. Moreover, it should be emphasized that the MC generated datasets are prone to statistical fluctuations. Therefore, a fast and accurate method for generation of raw data taking into account the BHE effect for evaluation of BHC algorithms is highly desired.

In this paper, we propose a hybrid approach combining MC and analytical simulations to speed up x-ray CT modeling. In this method, the contribution of the primary component to the projections is calculated through analytical simulations using ray-tracing methods whereas the contribution of the scatter component is calculated using pure MC simulations. The final projections are determined by appropriate combination of both simulation results. In addition, an analytic method for modeling of BHE in CT is proposed for fast generation of x-ray CT raw data.

II. MATERIALS AND METHODS

A. Hybrid Simulation

Our recently developed MCNP4C-based Monte Carlo x-ray CT simulator [5] for modeling multi-slice CT scanners was used to calculate the contribution of the scatter component for a cylindrical water and polyethylene phantoms using pure MC simulations. The MC simulator was validated through comparison with experimental measurements of different nonuniform phantoms with varying sizes on a clinical single-slice GE HiSpeed X/iF CT scanner (GE Healthcare Technologies, Waukesha, WI). The MC simulator is based on the MCNP4C code, a general-purpose, continuous-energy, generalized-geometry, time-dependent, coupled neutron/photon/electron Monte Carlo transport code [6]. Since MCNP is not capable of simulating gantry rotation, the geometry of each view is created in separate files using the developed user interface running under Matlab 7.4.0 (The MathWorks Inc., Natick, MA, USA). The scatter contribution during the simulation of CT data was separated from the primary component using the surface source method implemented in the MCNP4C Monte Carlo code. In this method, a virtual plane is considered after the scatter medium (phantom) and the direction, energy and history of each photon passing through this plane is registered. In the next step, the scatter component is calculated by considering only photons which had at least one interaction before hitting this plan toward the detection system. The unscattered component is calculated using the same method by considering only photons which had no interaction before hitting the virtual plane.

The contribution of the primary component to the projections is calculated through analytical simulation based on ray-tracing methods [7] using code written in Matlab 7.4.0.

Beam hardening correction, usually integrated in commercial software supplied by scanner manufacturers, was performed on the pure MC projections according to the method described by Kanamori *et al* [8].

The final projections in the hybrid simulator are determined by appropriate combination of MC and analytic simulation results as follow: The attenuation profile in the object can be calculated using equation 1:

$$\int_0^I \mu(x) dx = \text{Ln}\left(\frac{I_0}{I}\right) \quad (1)$$

Where I_0 and I are the number of detected photons for a blank scan and a scan with the object in the field of view, respectively. The blank scan and object scan can be divided in two parts as shown in equation 2.

$$I = P + S \quad I_0 = P_0 + S_0 \quad (2)$$

Where P and S represent primary and scattered components, respectively. By plugging equation 2 into equation 1:

$$\text{Ln}\left(\frac{I_0}{I}\right) = \text{Ln}\left[\frac{P_0(1+S_0/P_0)}{P(1+S/P)}\right] = \text{Ln}\left(\frac{P_0}{P}\right) + \text{Ln}\left(\frac{1+S_0/P_0}{1+S/P}\right) \quad (3)$$

Since the contribution of scattered photons in the blank scan is very small ($S/P \gg S_0/P_0$), equation 3 can be rewritten as:

$$\text{Ln}\left(\frac{I_0}{I}\right) = \text{Ln}\left(\frac{P_0}{P}\right) + \text{Ln}\left(\frac{1}{1+S/P}\right) \quad (4)$$

where the term $\text{Ln}(P_0/P)$ reflects the contribution of primary radiation which can be calculated by analytic simulation whereas the term S/P reflects the contribution of scattered radiation and can be calculated from pure Monte Carlo simulations.

B. Analytic Modeling of BHE

For analytic modeling of BHE, the contribution of each photon's energy in the x-ray spectrum can be calculated using equation 5.

$$k(E_i) = \frac{N_i}{N_0} \quad (5)$$

Where N_0 is the total number of photons in the x-ray spectrum and N_i is the number of photons in the i th energy bin. So the total number of photons in the spectrum can be re-written as:

$$N_0 = N_0 \times \sum_{i=1}^n k(E_i) \quad (6)$$

$$\sum_{i=1}^n k(E_i) = 1$$

Suppose I is the number of photons that hit each detector element (u, v) then:

$$I(u, v) = \sum_{i=1}^n I(u, v, E_i) \quad (7)$$

If we also suppose that I_0 is the number of detected photons in the blank scan, the number of detected photons in each energy bin can be calculated from equation 8.

$$I_0(u, v, E_i) = I_0(u, v) \times k(E_i) \quad (8)$$

Then the number of photons for the patient scan can be calculated from equations 9 and 10.

$$I(u, v, E_i) = I_0(u, v, E_i) \times \text{Exp}[-\mu(E_i) \times d] \quad (9)$$

$$I(u, v) = I_0(u, v) \sum k(E_i) \times \text{Exp}[-\mu(E_i) \times d] \quad (10)$$

Finally, the attenuation profile can be analytically calculated from equation 11.

$$AP(u, v) = -\text{Ln} \frac{I(u, v)}{I_0(u, v)} = -\text{Ln} \left[\sum_{i=1}^n k(E_i) \times \text{Exp}[-\mu(E_i) \times d] \right] \quad (11)$$

The calculated attenuation profile using this method includes the beam hardening effect. The contribution of scattered radiation can be added to the calculated attenuation profile to generate a realistic profile (i.e. including primary and scatter radiation as well as beam hardening effect). The whole process for modeling of BHE in all projections (around 1000) and also small x-ray energy bins (around 240 bin, each 0.5 keV) for generation of a sinogram corresponding to a cylindrical water phantom with this method takes less than 10 minutes using a standard Pentium IV PC. It should be emphasized that the proposed method can be used for fast generation of attenuation profiles for more complex phantoms.

III. RESULTS

Fig. 1 shows the comparison between water and polyethylene cylindrical phantom attenuation profiles before and after beam hardening effect correction using the method proposed by Kanamori *et al.* [8]. The underestimation of the attenuation profile before BHE removal shows the contribution of BHE in the attenuation profiles. It should be noted that the original profile was calculated using pure Monte Carlo simulations for the geometry of the single-slice GE HiSpeed X/iF CT scanner. A total number of 1.99×10^9 photons at 120 kVp were simulated in order to generate the attenuation profiles with less than 0.5% statistical uncertainty.

Figure 2 illustrates the excellent agreement between attenuation profiles of a uniform cylindrical water phantom and polyethylene phantom containing centered steel rod insert generated using pure Monte Carlo calculations based on our

MCNP4C-based MC simulator and hybrid simulations combining analytical and MC calculations.

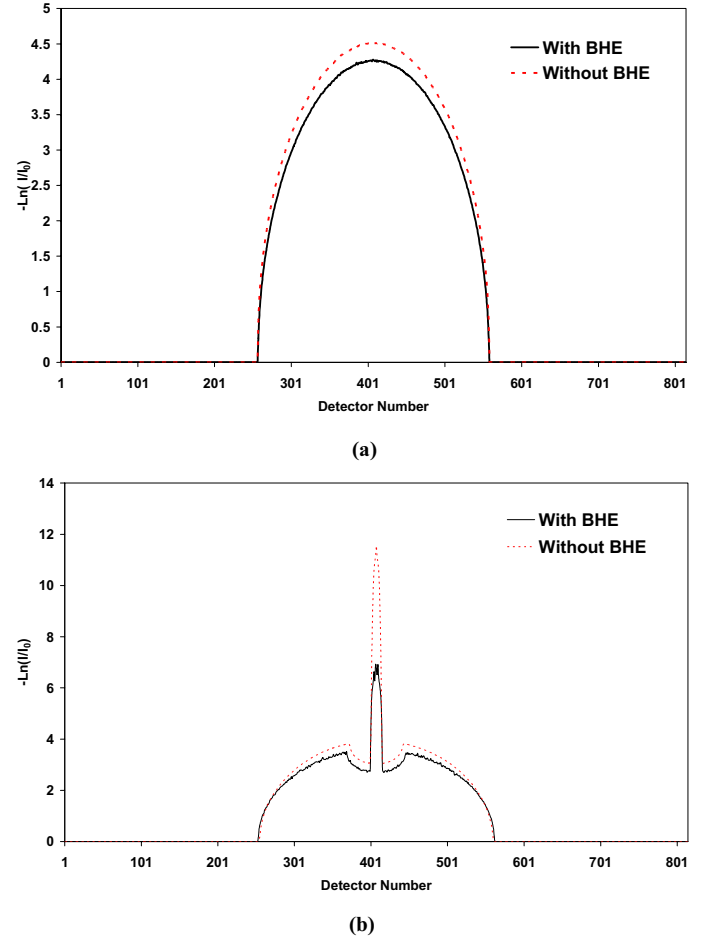


Fig. 1. Comparison of attenuation profiles for a uniform cylindrical water phantom (a) and polyethylene phantom containing centered steel rod inserts (b) before and after beam hardening correction.

Likewise, the images of the physical and simulated water and polyethylene phantoms reconstructed using a filtered backprojection algorithm implemented in commercial software (incorporating beam hardening and scatter correction) and Matlab reconstruction routine after beam hardening correction are shown in Figure 3. There is good agreement between the simulated and measured projections and reconstructed images.

Figure 4a shows the comparison of simulated attenuation profile from a cylindrical water phantom when using the x-ray spectra at 120 kVp including the BHE using pure Monte Carlo simulation and proposed analytical model. There is good agreement between pure Monte Carlo and analytical modeling.

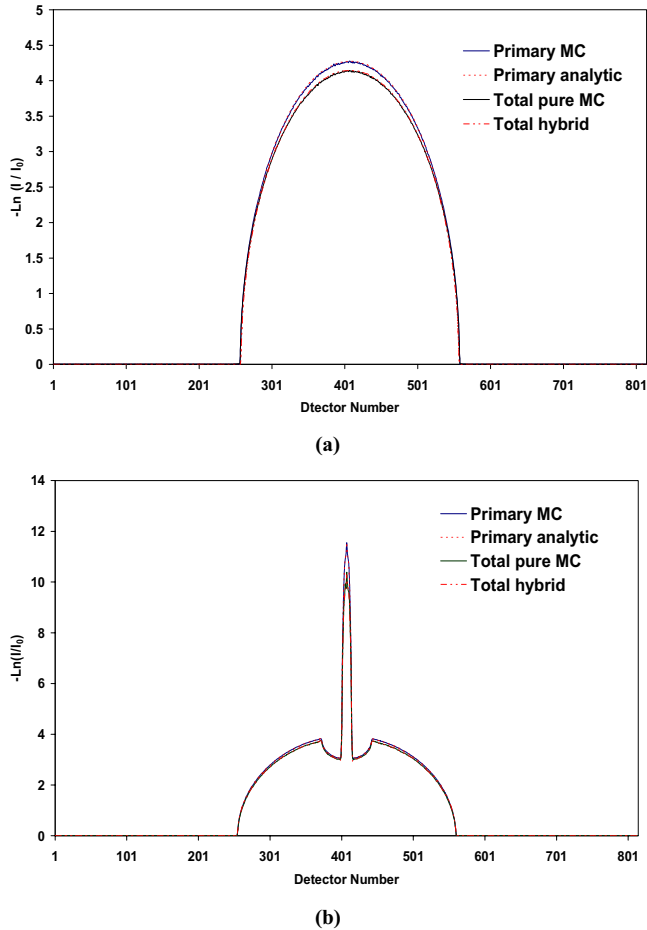


Fig. 2. Comparison of attenuation profiles for a uniform cylindrical water phantom (a) and polyethylene phantom containing centered steel rod insert (b) computed using pure MCNP4C-based Monte Carlo calculations and hybrid simulations combining analytical and Monte Carlo calculations.

Figure 5a and 5b shows the reconstructed water phantom image from simulated data when using the projections simulated by analytical model at 120 kVp x-ray spectrum and also 70 keV mono energetic x-ray photons (the effective energy of 120 kVp x-ray spectrum for single-slice GE HiSpeed X/iF CT scanner) in order to show the impact of BHE in the reconstructed image. It should be emphasized that the simulation of scattered radiation didn't consider due to the fact we wanted to remove the cupping artifact arising from the contribution of scattered radiation in the simulated profiles. Figure 5c shows a horizontal profile from the reconstructed images. The cupping artifact if the simulated image with BHE is obvious in the related profile.

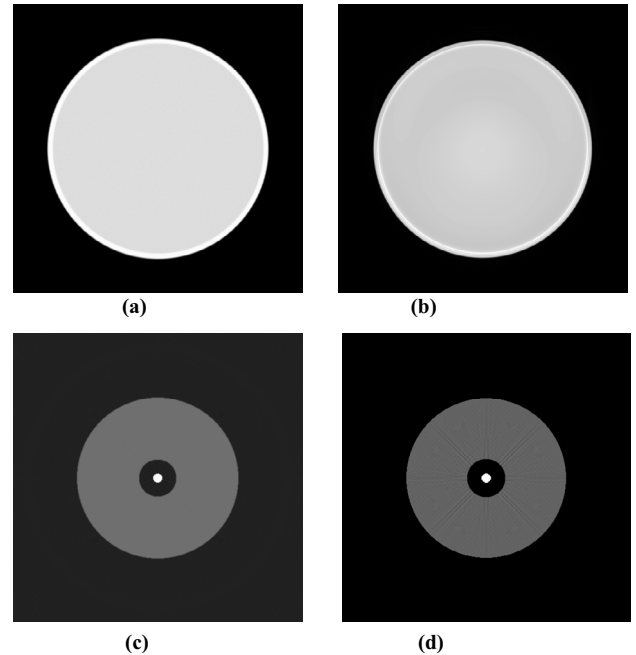


Fig. 3. Representative images of the physical cylindrical water phantom reconstructed using commercial software (a) and simulated cylindrical water phantom using hybrid method reconstructed using filtered backprojection Matlab routine (b). Same as above for the inhomogeneous polyethylene phantom illustrating measured (c) and simulated using the hybrid approach (d).

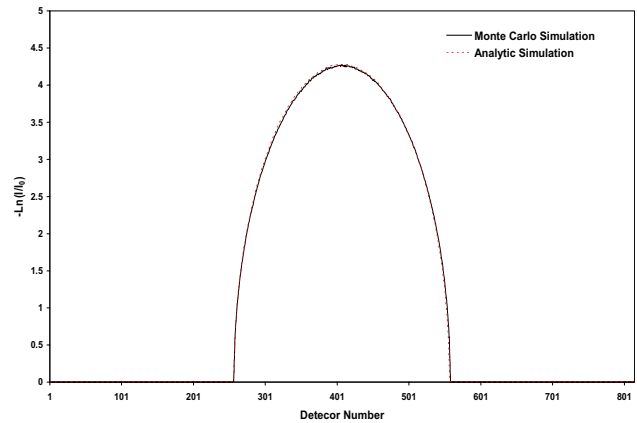


Fig. 4. Comparison of attenuation profiles for a uniform cylindrical water phantom using pure MC simulation and analytical method for assessment of BHE.

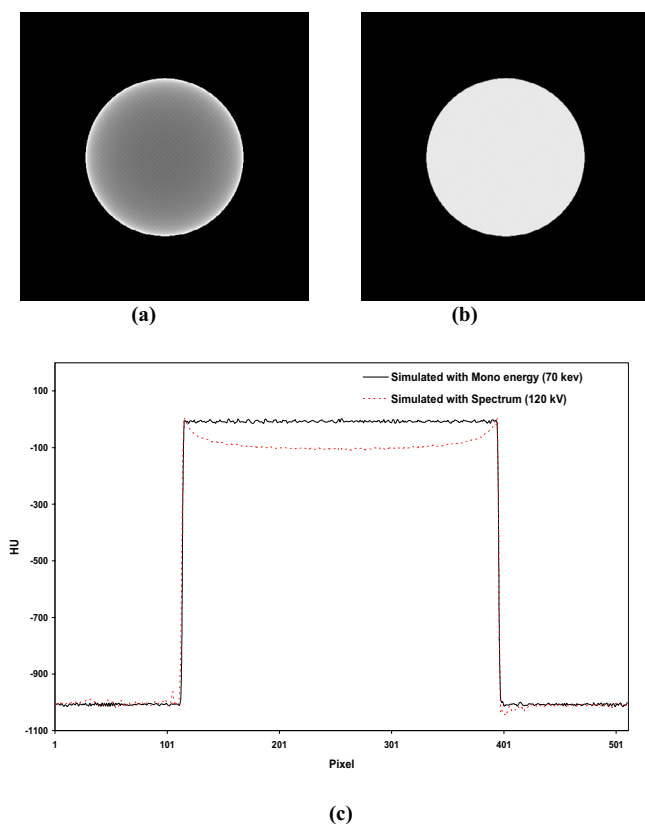


Fig. 5. Representative images of the cylindrical water phantom simulated using pure analytic method when using x-ray spectrum for assessment of BHE reconstructed using filtered backprojection Matlab routine (a) and simulated cylindrical water phantom using pure analytic method when using mono energy source (b). The horizontal profile of reconstructed image acquired using monoenergetic x-ray beam and spectrum by analytic simulation. The cupping artifact is obvious in the image acquired with spectrum that shows the effect of BHE (c).

IV. DISCUSSION

X-ray computed tomography (CT) is a medical imaging modality that produces cross-sectional images representing the x-ray attenuation properties of the body. Despite worthwhile research efforts and achievements in instrumentation, x-ray CT images still have the inherent tendency to produce physics-related artifacts owing to the fact that the images are reconstructed from a large number of independent detector elements. There are several sources of error and artifact that affect x-ray CT image quality. The assessment of their relevance is generally commended with the aim to reduce their impact either by optimizing the scanner design or by devising appropriate image correction and reconstruction strategies.

The evaluation of the effect of physical, geometrical and other design parameters on scanner performance and resulting image quality and patient dose could be achieved through cumbersome experimental measurements using developed test prototypes [9] or more conveniently using deterministic mathematical modeling [10] or sophisticated Monte Carlo (MC) simulations [5]. The latter relies on either in-house

developed dedicated programs [11] or public domain general purpose MC codes such as MCNP [5], EGS4 [12] and GEANT4 [13]. Nowadays, the MC method is widely used for solving many scientific problems involving statistical processes and is particularly well suited for medical physics and biomedical engineering applications due to the stochastic nature of radiation emission, transport and detection processes. The general idea of MC analysis is to create a model as similar as possible to the real system under study and calculate the interaction within the modeled system based on known probabilities of occurrence using random sampling of probability density functions for each event.

One significant problem in the use of MC calculations is the presence of statistical uncertainties (noise) in the estimates. A simple but not practical way to decrease statistical uncertainties is to run MC simulations for sufficiently long time (large number of histories) and use efficient variance reduction techniques. Currently, two approaches are used to reduce statistical uncertainties from MC calculations: hardware (parallelization) and software (de-noising) approaches. It should be noted that optimization and validation of these approaches is still an area of considerable research interest that requires further research and development efforts. More recently, combination of pure Monte Carlo simulations and analytical calculations has been proposed for modeling of planar x-ray imaging. In this paper, we present a new combined method called *hybrid simulation* for modeling x-ray CT scanners. The method was validated through comparison to experimental measurements and pure Monte Carlo generated datasets corresponding to a single-slice CT scanner. Nevertheless, the technique can be easily extended for modeling multi-slice CT scanners. The achieved speed up factor using the hybrid method is the result of the use of analytic simulation of primary photons using well established ray-tracing algorithms. On the other hand, modeling the scatter component using pure Monte Carlo calculations is fast enough since a smaller number of photon histories ($10E+6$ instead of $1.99E+9$) is required for calculation of scatter to primary ratio (SPR) with acceptable accuracy in a limited number of projections (around 180 instead of 1000, depending on the phantom complexity).

The differences observed between the attenuation profiles in figure 1 before and after BHE removal are due to the fact that owing to BHE, the estimated linear attenuation coefficients of various materials are less than the actual values since attenuation coefficients are energy-dependent and decrease with increasing of photons' energy. Thus the calculated attenuation profile including the BHE is overestimated as shown in figure 1.

Figure 2 shows excellent agreement between the attenuation profiles calculated using pure MC simulation and hybrid simulation. The calculation of primary radiation took advantage of the surface source utility in the MCNP4C code whereas for analytic calculation of primary radiation the ray-tracing method was used. The linear attenuation coefficients at each energy bin were calculated using the XCOM photon

cross section libraries [14]. The total attenuation profile shown in figure 2 obtained by combining analytic and MC methods has lower intensity than the primary profile because of the contribution of scattered radiation in the final profile which indeed underestimates attenuation coefficients during the CT image reconstruction process. This should ideally be corrected during the reconstruction process.

Figure 3 shows the good agreement between the reconstructed images from both phantoms using simulated and measured attenuation profiles. It should be noted that the measured data are reconstructed using commercial software whereas the simulated data are reconstructed using a Matlab's reconstruction routine after beam hardening removal from the projections.

Figure 4 and 5 further validate analytic BHE modelling in comparison with MC simulation data. Analytic modelling of BHE simplifies greatly the simulation process and will be particularly useful for generation of raw data for testing novel beam hardening correction algorithms.

V. CONCLUSION

In order to speed-up x-ray CT modeling, a hybrid approach combining MC and analytical simulations, has been proposed. In this approach, the contribution of the primary component to the projections is calculated through analytical simulations using ray-tracing methods whereas the contribution of the scatter component is calculated using pure MC simulations. The final projections are determined by appropriate combination of both simulation results. The hybrid simulator was validated through comparison with experimental measurements and pure MC simulations of different phantoms performed on clinical CT scanners. There is good agreement between the simulated and measured projections and reconstructed images. This work has demonstrated the feasibility of using the hybrid approach for fast and accurate modeling of CT scanners thus allowing to evaluate the effect of physical, geometrical and other design parameters on CT scanner performance and image quality. The technique is being improved by incorporating fast ray-tracing algorithms and also being compared to other denoising techniques developed specifically to reduce statistical fluctuations in MC estimates.

REFERENCES

[1] J. Hsieh, "Analytical models for multi-slice helical CT performance parameters," *Med Phys*, vol. 30, pp. 169-178, 2003.
 [2] H. Zaidi and M. R. Ay, "Current status and new horizons in Monte Carlo simulation of X-ray CT scanners," *Med Biol Eng Comput*, vol. 45, pp. 809-817, 2007.
 [3] P. Duvauchelle, N. Freud, V. Kaftandjian, and D. Babot, "A computer code to simulate X-ray imaging techniques.," *Nucl Instrum Methods Phys Res B*, vol. 170, pp. 245-258, 2000.

[4] B. De Man, J. Nuyts, P. Dupont, G. Marchal, and P. Suetens, "Reduction of metal streak artifacts in X-ray computed tomography using a transmission maximum a posteriori algorithm," *IEEE Trans Nucl Sci*, vol. 47, pp. 977-981, 2000.
 [5] M. R. Ay and H. Zaidi, "Development and validation of MCNP4C-based Monte Carlo simulator for fan- and cone-beam x-ray CT," *Phys Med Biol*, vol. 50, pp. 4863-4885, 2005.
 [6] J. F. Briesmeister, "MCNP - A general Monte Carlo N-particle transport code. version 4C," Los Alamos National Laboratory, NM, Los Alamos, NM LA-13709-M, 2000.
 [7] H. Zhao and A. J. Reader, "Fast ray-tracing technique to calculate line integral paths in voxel arrays," in *2003 IEEE Nuclear Science Symposium Conference Record*, 2003, pp. 2808-2812.
 [8] H. Kanamori, N. Nakamori, K. Inoue, and E. Takenaka, "Effect of scattered x-ray on CT images," *Phys Med Biol*, vol. 30, pp. 239-249, 1985.
 [9] S. C. Lee, H. K. Kim, I. K. Chun, M. H. Cho, and S. Y. Lee, "A flat-panel detector based micro-CT system: performance evaluation for small-animal imaging," *Phys Med Biol*, vol. 48, pp. 4173-4185, 2003.
 [10] J. H. Siewerdsen and D. A. Jaffray, "Optimization of x-ray imaging geometry (with specific application to flat-panel cone-beam computed tomography)" *Med Phys*, vol. 27, pp. 1903-1914, 2000.
 [11] A. P. Colijn, W. Zbijewski, A. Sasov, and F. J. Beekman, "Experimental validation of a rapid Monte Carlo based micro-CT simulator," *Phys Med Biol*, vol. 49, pp. 4321-4333, 2004.
 [12] M. Caon, G. Bibbo, and J. Pattison, "A comparison of radiation dose measured in CT dosimetry phantoms with calculations using EGS4 and voxel-based computational models," *Phys Med Biol*, vol. 42, pp. 219-29, 1997.
 [13] M. Khodaverdi, A. F. Chatzioannou, S. Weber, K. Ziemons, H. Halling, and U. Pietrzyk, "Investigation of different microCT scanner configurations by GEANT4 simulations," *IEEE Trans Nucl Sci*, vol. 52, pp. 188-192, 2005.
 [14] M. J. Berger and J. H. Hubbell, "XCOM: Photon cross sections on a personal computer," National Bureau of Standards (US) NBSIR 87-3597, 1987.